Cyclostationarity based blind block timing estimation for alamouti coded mimo signals

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Cyclostationarity Based Blind Block Timing Estimation for Alamouti Coded MIMO Signals

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***Abstract*—Blind parameter estimation algorithms provide a powerful tool for application scenarios where the use of training or pilot sequences is not desirable, e.g. in order to improve the bandwidth efficiency of the transmission, or in non-cooperative scenarios where such sequences are not available to the receiver. This work proposes a blind block timing estimation algorithm for Alamouti space-time block coded signals exploiting the second order joint cyclostationary characteristics of the received signal vector, which is induced by the space time block coding operation performed by the transmitter. The proposed algorithm outperforms the existing algorithms by a wide margin.**

***Keywords*—Alamouti space-time block code, MIMO, Blind parameter estimation, Timing synchronization**

I. INTRODUCTION

Space-time block coding is a transmission technique extensively employed in wireless Multiple Input Multiple Output (MIMO) systems for improving the reliability of the transmission by exploiting the spatial diversity inherent to the multiantenna communications. This improvement is achieved by encoding and multiplexing blocks of modulated transmit symbols across multiple transmit antennas and time slots using so called space-time block codes (STBC), inducing a space-time redundancy in the transmit signal, which is then exploited at the receiver side, providing diversity gain.

The recovery of the transmitted information symbols in a space-time block coded MIMO system at the receiver side requires the estimation of several parameters, such as the channel state, the carrier frequency offset, symbol timing, and additionally, the block timing, which is a parameter specific to the STBC-MIMO communications. While these parameters may be estimated in a data-aided manner using pre-arranged training symbols, there exists application scenarios, where the use of such methods may not be desirable. Conventionally, concerns on the efficient use of the available bandwidth have been the main motivation for using non-data-aided or blind parameter estimation algorithms instead of their data-aided counterparts. Furthermore, data-aided parameter estimation is not suitable in scenarios, where cooperation between the receiver and transmitter is not possible or not desirable.

This work investigates the problem of the blind block timing estimation in a space-time block coded MIMO system. The block timing, which refers to the time instants where each received encoded symbol block begins, is an essential parameter required to properly decode and demodulate the space time block coded transmit signal blocks at the MIMO receiver, hence, its blind estimation is of paramount practical interest. In [1], a blind block timing synchronization algorithm is proposed for an Alamouti coded MIMO signal in a Nakagami-m fading channel employing a test statistic based on the fourth order moments of the received signal. While this algorithm has the advantage of requiring a receiver with only one single receive antenna, its performance is only adequate for large values of the shape parameter m of the Nakagami-m distributed channel. Thus, it is not suitable for Rayleigh fading, which can be considered as a special case of Nakagami-m fading with $m = 1$. In [2], a blind block-timing and carrier frequency offset estimation algorithm has been proposed for the class of orthogonal STBCs, which is based on the second order statistics of the signal received by a two-antenna receiver.

In this work, we propose a block timing estimation method for a MIMO system employing the well known and widely used Alamouti STBC [3]. The proposed algorithm exploits the second order joint cyclostationary characteristics of the transmit signal vector induced by the coding operation and outperforms the existing methods in the literature, without requiring any a-priori information of the channel state, signal to noise ratio (SNR) and the employed modulation type, which makes the algorithm especially suitable for non-cooperative application scenarios and applications involving MIMO signal identification that have recently become a focus of intensive research efforts (see, for example [4], [5] and [6]).

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a MIMO system with 2 transmit and N_r receive antennas. The Alamouti encoder maps the k'th modulated information bearing symbol block of length 2 $\mathbf{x}[k] = [x_0[k], x_1[k]]$ into a 2×2 encoded symbol block $\mathbf{C}(\mathbf{x}[k])$ to be transmitted in 2 time slots, such that

$$\mathbf{C}(\mathbf{x}[k]) = \begin{bmatrix} x_0[k] & -x_1^*[k] \\ x_1[k] & x_0^*[k] \end{bmatrix}. \tag{1}$$

The components of the transmitted signal vector at the n'th time slot $\mathbf{y}[n] = [y_0[n], y_1[n]]^T$ can be expressed as:

$$y_0[n] = \sum_{k=-\infty}^{\infty} x_0[k]\delta(n-2k) - x_1^*[k]\delta(n-(2k+1)) \tag{2}$$

and

$$y_1[n] = \sum_{k=-\infty}^{\infty} x_1[k]\delta(n-2k) + x_0^*[k]\delta(n-(2k+1)), \tag{3}$$

where $\delta[n]$ is the unit impulse function. Assuming a receiver with N_r receive antennas, and a flat fading channel, the signal component at the i'th receive antenna can be given as

$$r_i(t) = \sum_{v=0}^{1} h_{v,i} \sum_{n=-\infty}^{\infty} e^{j\varphi} y_v[n] p(t-nT-\Delta) + w_i(t), \tag{4}$$

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where $h_{v,i}$ is the complex channel coefficient between the v'th transmit and i'th receive antenna, φ is the carrier phase offset, $p(t)$ is the impulse response of the cascade of the transmit and receive filters, T is the symbol period, Δ is the timing offset, $w_i(t)$ is the contribution of the noise at the i'th receive antenna. Since the Alamouti coded signal blocks have a length of 2 time slots, the timing offset parameter Δ can be expressed, without loss of generality, as $\Delta = (l + \varepsilon)T$, where the integer part $l \in \{0, 1\}$ represents the block timing and the fractional part $0 < \varepsilon < 1$ is the symbol timing offset. Initially, we will assume that the estimation of the symbol timing has been performed previously, and its effect has been compensated. The effect of any residual symbol timing error on the proposed block timing estimation algorithm will be investigated later in the paper via simulations. Under these assumptions the signal vector at the receiver after symbol-rate sampling is given as:

$$\mathbf{r}[n] = \mathbf{H}\mathbf{y}[n-l] + \mathbf{w}[n]\ , \tag{5}$$

where $\mathbf{H}$ is the $N_r \times 2$ channel matrix which incorporates the channel coefficients $h_{v,i}$ and the oscillator phase offset φ, and $\mathbf{w}[n]$ is the noise vector. Since $l \in \{0, 1\}$, the estimation of the code block timing reduces to a binary hypothesis test. In this work, we propose a block timing estimation technique based on the joint cyclostationary characteristics of the transmit signal induced by the space time block coding operation.

III. Cyclostationarity of the Transmit Signal

Two complex valued discrete time random processes $u[n]$ and $v[n]$ are referred to as jointly wide-sense cyclostationary if either their cross correlation function $R_{uv}[n, \tau] = E\{u[n]v^*[n-\tau]\}$ or their conjugate cross correlation function $R_{uv^*}[n, \tau] = E\{u[n]v[n-\tau]\}$ (or both) exhibits a periodicity in the time index n [7], [8]. Clearly, such a periodicity allows the following Fourier series representation for these functions:

$$R_{uv}[n, \tau] = \sum_{\alpha = 2\pi m/K} R^{\alpha}_{uv}[\tau] e^{j\alpha n}, \tag{6}$$

and

$$R_{uv^*}[n, \tau] = \sum_{\alpha = 2\pi m/L} R^{\alpha}_{uv^*}[\tau] e^{j\alpha n}, \tag{7}$$

where K and L are the fundamental periods of the respective crosscorrelation functions, and the sums are taken over the integer multiples of the fundamental cyclic frequencies $\alpha_0 = 2\pi/K$ and $\alpha'_0 = 2\pi/L$ respectively. The Fourier series coefficients, $R^{\alpha}_{uv}[\tau]$ and $R^{\alpha}_{uv^*}[\tau]$, which depend on the cycle frequency parameter α and the lag parameter τ are referred to as the cyclic crosscorrelation function (CCF) and the conjugate cyclic crosscorrelation function (CCCF) respectively.

Using the equations (2) and (3) with the assumption of an uncorrelated information bearing data sequence, it can be shown that the conjugate cross correlation function $R_{y_0 y_1^*}[n, \tau]$ of the components of the transmit signal vector, $\mathbf{y}[n-l] = [y_0[n-l], y_1[n-l]]^T$ in (5) is periodic in n with a period $L = 2$, exhibiting a joint conjugate cyclostationary characteristic. The corresponding CCCF depends on the value of the block timing parameter $l \in \{0, 1\}$ and can be derived by substituting (2) and (3) in equation (7) and calculating the corresponding Fourier Series coefficients:

$$R^{\alpha = m\pi}_{y_0 y_1^*}[\tau] = \begin{cases} \frac{1}{2}, & \tau = -1 \\ \frac{1}{2}(-1)^{m-1}, & \tau = 1 \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

for $l = 0$, whereas

$$R^{\alpha = m\pi}_{y_0 y_1^*}[\tau] = \begin{cases} \frac{1}{2}(-1)^{m}, & \tau = -1 \\ -\frac{1}{2}, & \tau = 1 \\ 0, & \text{otherwise}, \end{cases} \tag{9}$$

for $l = 1$. In this work, we propose to exploit this dependency in the extraction of the block timing parameter l.

IV. Proposed Algorithm

It should be noted that at the receiver, the signal $\mathbf{y}[n-l]$ is not available, and although the components of the received signal $\mathbf{r}[n]$ also exhibit joint wide sense cyclostationarity with the same cyclic frequency as the transmit signal, the extraction of the block timing parameter from the CCCF of its components is far from trivial, since each of those components contains a linear mixture of the components of the transmit signal. Here, we employ an alternative approach by blindly compensating the channel effects using a blind source separation (BSS) technique in order to recover a noisy version of the transmit signal vector with channel estimation ambiguities, and exploit its cyclostationary characteristics for the block timing estimation.

A. Blind Estimation and Compensation of the Channel Matrix

BSS techniques are computational methods employed for blindly separating linear mixtures of discrete time random processes into their individual components [9]. In this work, we propose to use the joint approximate diagonalization of eigenmatrices (JADE) algorithm, which is a popular BSS method based on the minimization of the nondiagonal elements of the fourth order cumulant tensor of the whitened input vector sequence [10], [11]. In [11], it has been shown that, despite the fact that it has been designed with independent signal components in mind, the JADE algorithm exhibits a robustness to the time correlations between the transmit signal components induced by the space time block coding operations, including the Alamouti and other orthogonal- and quasi-orthogonal codes. Furthermore, compared to other existing algorithms for blind channel matrix estimation in STBC-MIMO systems, JADE has the advantage of requiring a considerably less amount of a-priori information on the signal and can be employed for a wider range of modulation types and space-time block codes (see [11] for a detailed comparison).

Following the generation of the channel matrix estimate $\hat{\mathbf{H}}$ blindly using the JADE algorithm, the transmit signal vector is estimated as:

$$\hat{\mathbf{y}}[n-l] = (\hat{\mathbf{H}}^{\dagger}\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^{\dagger}\mathbf{r}[n]. \tag{10}$$

It should be noted that the JADE algorithm inherently contains phase and permutation ambiguities [9], i.e. the estimate of the channel matrix $\hat{\mathbf{H}}$ generated with JADE can be written as (neglecting the estimation errors):

$$\hat{\mathbf{H}} = \mathbf{H}\mathbf{P}\mathbf{\Theta}, \tag{11}$$

where $\mathbf{P}$ is a random 2×2 permutation matrix and $\mathbf{\Theta}$ is a 2×2 diagonal matrix where $[\mathbf{\Theta}]_{k,k}=e^{j\phi_k}$ with random phases ϕ_k, $k=0,1$. Thus, the components of the estimated transmit signal vector $\hat{\mathbf{y}}[n-l]$ contain random phase rotations and their order is randomly interchanged, both of which affect the values of the CCCF function of the components of $\hat{\mathbf{y}}[n-l]$, rendering the direct use of these functions as test statistics impossible. Instead, we propose employing a decision statistics based on the CCCF functions of the estimated transmit signal vector which is invariant to the phase and permutation ambiguities.

B. The Decision Statistics

It is straightforward to show that the random phase offsets ϕ_0 and ϕ_1 introduced by JADE in the estimated signal components $\hat{y}_0[n-l]$ and $\hat{y}_1[n-l]$ affect a random phase shift in the corresponding CCCF function, i.e.

$$R^{\alpha}_{\hat{y}_0\hat{y}_1^*}[\tau]=e^{j(\phi_0+\phi_1)}R^{\alpha}_{y_0y_1^*}[\tau]. \tag{12}$$

Similarly, it can be shown that due to the code structure, the presence of the permutation ambiguity leads to a random sign change in the CCCF, which can also be considered as an additional phase shift of the CCCF function by an amount of π. Clearly, these phase shifts are independent of the cycle frequency α, and this independence can be exploited to form test statistics invariant to the phase and permutation ambiguities introduced by the blind channel estimation.

Let q_τ be the ratio of the value of $R^{\alpha}_{\hat{y}_0\hat{y}_1^*}[\tau]$ at $\alpha=0$ and π. Clearly this ratio is invariant to the effects of the phase and the permutation ambiguities discussed above, i.e., using (12):

$$q_\tau=\frac{R^{\alpha=0}_{\hat{y}_0\hat{y}_1^*}[\tau]}{R^{\alpha=\pi}_{\hat{y}_0\hat{y}_1^*}[\tau]}=\frac{R^{\alpha=0}_{y_0y_1^*}[\tau]}{R^{\alpha=\pi}_{y_0y_1^*}[\tau]}. \tag{13}$$

Furthermore, this ratio depends directly on the value of the block timing parameter l, which can be seen by substituting (8), (9) and (12) in (13). Thus, q_τ can be employed as a decision statistics for the estimation of l. Let $\mathbf{q}=[q_{-1},q_1]^T$ be the vector obtained by stacking the values of q_τ at $\tau=-1$ and 1 respectively. Using (8), (9), (12) and (13), the binary hypothesis test for the estimation of the block timing parameter l can be formulated as:

$$\begin{aligned} H_0:&\quad l=0,\mathbf{q}=[1,-1]^T,\\ H_1:&\quad l=1,\mathbf{q}=[-1,1]^T, \end{aligned} \tag{14}$$

regardless of the presence of the phase and permutation ambiguities. The decision for the block timing can be performed using an estimate of this vector, $\hat{\mathbf{q}}=[\hat{q}_{-1},\hat{q}_1]^T$ by replacing the actual values of the CCCF in Eq. (13) with their finite sample estimates,

$$\hat{R}^{\alpha}_{\hat{y}_0\hat{y}_1^*}[\tau]=\frac{1}{N}\sum_{n=0}^{N-1}\hat{y}_0[n-l]\hat{y}_1[n-l-\tau]e^{-j\alpha n}, \tag{15}$$

which are asymptotically (i.e. as the sample size $N\to\infty$) unbiased and consistent estimates and their probability density converges to a complex valued joint gaussian distribution [8].

The derivation of the optimal decision rule for this binary hypothesis testing problem requires the computation of the exact joint probability distribution of $\hat{q}_{-1}$ and $\hat{q}_1$, which is mathematically intractable due to the fact that these are essentially ratios of correlated complex valued nonzero mean asymptotically gaussian random variables, and the error distribution of the blind channel estimator is not available. Instead, we propose a simple decision approach based on the minimization of the euclidian distance $|\hat{\mathbf{q}}-\mathbf{q}|^2$ with respect to the two possible hypotheses. After some simple algebraic manipulations, this approach leads to the following decision rule for the block timing parameter:

$$Re\{\hat{q}_{-1}-\hat{q}_1\}\underset{\text{Decide for } H_1}{\overset{\text{Decide for } H_0}{\gtrless}}0, \tag{16}$$

with the real part operation $Re\{\}$.

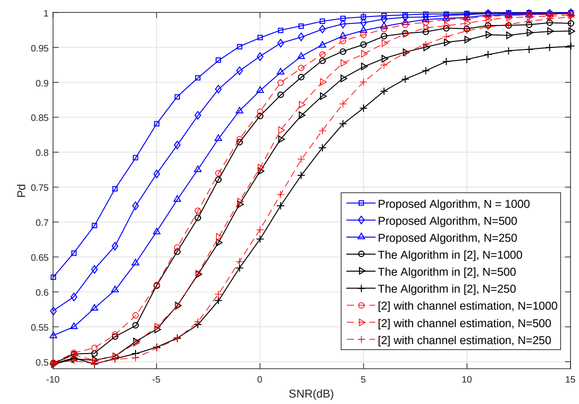

Fig. 1. Performance comparison of the proposed algorithm with the algorithm proposed in [2], N_r=2, N=250, 500 and 1000.

V. Simulation Results

In this section, the performance of the proposed block timing estimation method is evaluated via Monte Carlo simulations. An Alamouti coded STBC-MIMO transmitter is considered, which employs QPSK modulation in a frequency flat block fading channel, whose coefficients are modeled as zero-mean independent circular complex Gaussian random variables with unit variance. The average probability of correct block timing decision P_d is employed as a performance measure for the binary hypothesis testing problem, which, with the assumption of equiprobable hypotheses, can be written as $P_d=\frac{1}{2}\sum_{l=0}^{1}P(H_l|H_l)$, where $P(H_l|H_l)$ is the probability of correct decision for the hypothesis H_l. For each hypothesis, 10000 Monte Carlo trials have been performed per SNR value, which, with the assumption of unit power transmit signals, has been defined as SNR$=10\log_{10}(N_t/\sigma^2)$. Fig. 1 compares the performance of the proposed block timing estimation algorithm with that of [2] for the Alamouti STBC designed for a 2-antenna receiver. The simulations have been performed for $N=250$, 500 and 1000. In order to provide a fair comparison, $N_r=2$ has been chosen, and both the original version of the algorithm without any channel compensation, and a modified version with blind channel compensation using JADE have been considered. The simulation results show that the proposed

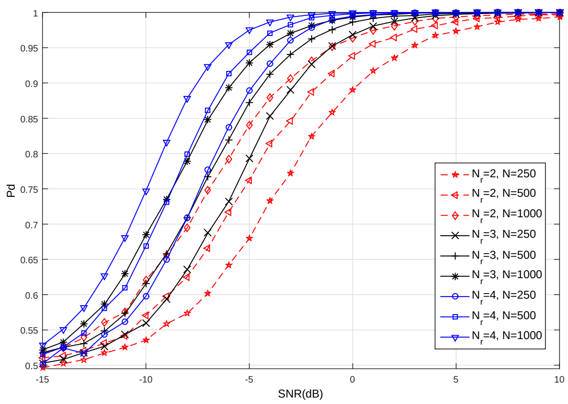

Fig. 2. Effect of N_r on the performance of the proposed algorithm. N_r=2, 3 and 4, N=250, 500 and 1000.

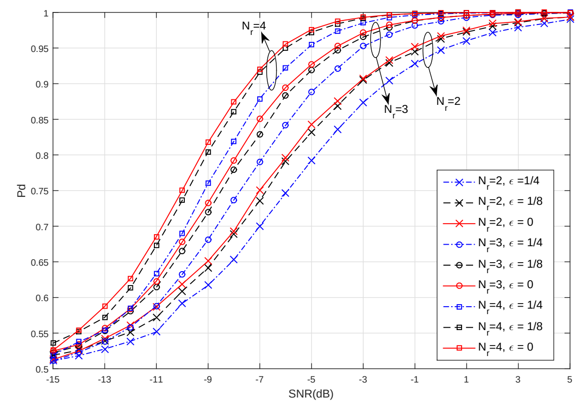

Fig. 3. Effect of the symbol timing error ε on the performance of the proposed algorithm for ε=0,1/8,and 1/4, N=1000, N_r=2,3 and 4.

algorithm outperforms the existing approach by a very wide margin in both cases. It should be noted that the computational cost of estimating the test statistics in (13) is the same as that of [2]. Thus, the proposed algorithm exhibits a higher computational complexity compared to [2], mainly due to the additional computational cost incurring from the blind channel estimation using the JADE algorithm.

Unlike [1] and [2], the proposed algorithm is not designed for a specific number of receive antennas and can employ any receiver with $N_r \geq 2$. Fig. 2 demonstrates the effect of N_r on the algorithm performance. $N_r = 2, 3$ and 4 is considered with $N = 250, 500$ and 1000. Apparently, increasing N_r improves the quality of the blind channel estimation, leading to an increase in the performance. As expected, both Figs. 1 and 2 show that the performance of the algorithm increases with increasing number of observed vector samples N.

Finally, Fig. 3 displays the effect of the normalized symbol timing error ε on the performance of the proposed block timing estimation algorithm. Here the employed pulse shape $p(t)$ in (4) is a raised-cosine with a roll off factor $\rho = 0.3$. $\varepsilon = 0$, 1/8 and 1/4 have been considered with $N = 1000$ and N_r =2, 3 and 4. The results indicate that the proposed algorithm displays a considerable robustness to symbol timing errors, exhibiting a performance loss of only around 1dB for ε =1/4 at $P_d = 0.95$ for each considered value of N_r.

VI. Conclusion

In this work, a novel blind block timing estimation algorithm for Alamouti coded MIMO signals has been presented, which exploits the joint wide sense cyclostationarity induced in the transmit signal by the space time block coding operation. The proposed method outperforms existing algorithms by a high margin, and does not require any a-priori information on the channel matrix, SNR and the modulation employed in the signal while exhibiting a robustness against symbol timing errors, which makes it especially suitable for non-cooperative application scenarios. The extension of the proposed approach to other existing space-time block codes and incorporation of blind frequency offset estimation into the algorithm are the subjects of our further research.

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